Ontology of Time in GFO

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Abstract. Time, events, changes and processes play a major role in conceptual modeling, and in information systems and computer science altogether. Accordingly, the representation of time structures and reasoning about temporal data and knowledge are important theoretical and practical research areas. We assume that a formal representation of temporal knowledge must use as a framework some top-level ontology that describes the most general categories of temporal entities.

In the current paper we discuss an ontology of time which is part of the foundational ontology GFO (General Formal Ontology). This ontology of time is inspired by ideas of Franz Brentano [1]. It is used to propose novel contributions to a number of problematic issues related to temporal representation and reasoning, among others, the Dividing Instant Problem and the problem of persistence and change. We present an axiomatization of the ontology as a theory in first-order logic. Eventually, metalogical analysis shows the consistency, completeness, and decidability of this theory.

Keywords. Ontology of Time, Time Point, Time Interval, Coincidence, Brentano

Introduction

Space and time are basic categories of any top-level ontology. They are fundamental assumptions for the mode of existence of those individuals that are said to be in space and time. In this paper we expound the ontology of time which is adopted by the General Formal Ontology (GFO) [2], a top-level ontology being developed by the Onto-Med research group[3] at the University of Leipzig. The time ontology together with the space ontology of GFO [3] forms the fundament for an ontology of material individuals.

There is an ongoing debate about whether time is ideal and subject-dependent or whether it is a real entity being independent of the mind. We defend the thesis that time exhibits two aspects. On the one hand, humans perceive time in relation to material entities through phenomena of duration, persistence, happening, non-simultaneity, order, past, present and future, change and the passage of time. We hold that these phenomena are mind-dependent. On the other hand, we assume that material entities possess mind-independent dispositions to generate these temporal phenomena. We call these dispositions *temporality* and claim that they unfold in the mind/subject as a manifold of tem-

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poral phenomena. This distinction between the temporality of material entities and the temporal phenomena (called *phenomenal time*) corresponds to the distinction between temporality and time as considered by Nicolai Hartmann in [4]. A satisfactory ontology of time and its formal representation should treat the various temporal phenomena in a uniform und consistent manner. We suggest to address the following basic tasks.

1. The development of an ontology of time itself, abstracted from real phenomena. We call this time *abstract phenomenal time*.
2. The development of an ontology, describing precisely how material entities (objects, processes) are related to the abstract phenomal time and how temporal phenomena are represented.
3. The establishment of a truth-relation between temporal propositions and spatio-temporal reality, also termed the problem of *temporal incidence* [5].
4. The elaboration of a unifying formal axiomatization of the ontology of time and of material entities of the spatio-temporal reality, the *formalization problem*.

The present paper is a contribution forward to their solution. While our results mainly pertain to the first and fourth issues, the second and third provide corresponding motivations and application cases.

The paper is organized as follows. In section 1 we present a focused state of art. Section 2 introduces selected problems of temporal qualification and incidence and proposes desiderata for an ontology of abstract phenomenal time. An informal introduction to time in GFO and to related kinds of entities in section 3 allows us in section 4 to illustrate new approaches to the problems previously introduced. Section 5 provides the axiomatization of abstract phenomenal time for GFO in first-order logic (FOL), accompanied by investigating the metalogical properties of the theory in section 6. In section 7 we draw some conclusions from our results and outline various problems and tasks for future research.

1. Time in Logic, Artificial Intelligence, and Ontologies

The literature on time and temporal representation and reasoning is vast. We merely point to some corresponding surveys [6,7,8] and strictly limit the scope of related work.

1.1. Axiomatic Theories of Time

Axiomatizations of time have been studied primarily in artificial intelligence. Recently, Lluís Vila surveyed time theories according to three major branches: purely point-based and purely interval-based theories, plus theories that combine points and intervals [5]. An earlier catalog of temporal theories is provided by Pat Hayes in [9]. Very recently, several of these and further theories have been implemented, verified and further formally analyzed regarding their metatheoretic relationships by Michael Grüninger et al., cf. [10].

The claimed focus of Vila's survey is on "the most relevant theories of time proposed in Artificial Intelligence according to various representational issues [. . .]" [5, p. 1], while its introduction links to many, frequently more specific works on time. For pure point-theories, Vila mainly presents typical axioms for a single relation *before* (time point x is properly before time point y): those of a linear order, unboundedness/infinity in both directions and the mutually exclusive axioms of discreteness (every time point has an immediate successor and predecessor) and density (between any two distinct time points there is a third one). This allows for interesting completeness results: if *before* is axiomatized as an unbounded, strict linear ordering and satisfies either discreteness or density, that yields a syntactically complete theory already [11].

In the case of purely interval-based theories, there is a similar result on an extended version of the interval theory of James F. Allen and Pat Hayes [12,13], referred to as $\mathcal{AH}$ in [5]. Allen had introduced a temporal interval algebra [14], cf. also [7, ch. 8], that is based on disjunctive combinations of the 13 well-known simple qualitative interval relations, including *equal*, *meets*, *before/after*, *starts/ends*, *overlap*, etc. Allen and Hayes then provided a FOL axiomatization of *meets* consisting of five axioms, and showed that the remaining 12 relations can be defined solely based on *meets*.[4] Together with an additional axiom enforcing a kind of density of the meeting points of time intervals, Peter Ladkin presents an extension to $\mathcal{AH}$ which he proved to be complete [15].

Despite the valuable results on pure interval theories, they are frequently considered to be insufficient unless time points are reintroduced or reconstructed [5, sect. 1.6–1.7], [16]. Reconstruction is typically mathematical in nature, e.g. by defining points as maximal sets of intervals that share a common intersection. In contrast, theories have been proposed that genuinely consider time points and intervals on a par. One example is the theory $\mathcal{IP}$ by Lluís Vila [5, sect. 1.7.2], [17], which extends the axioms of the pure point theory (without fixing density or discreteness) with 7 axioms that naturally link points and intervals using the point-interval relations *begin* and *end* (time point x is the begin/end of interval y). $\mathcal{IP}$ enjoys interesting metalogical properties [5, p. 16–17]: firstly, every model of $\mathcal{IP}$ is completely characterized by an infinite set with an unbounded strict linear order on it; secondly, the theory $\mathcal{IP}_{\text{dense}}$, obtained by adding density of time points to $\mathcal{IP}$, is logically equivalent to Ladkin's complete extension of $\mathcal{AH}$.

In summary, these are well-established and important theories to which each new proposal should be related (cf. section 6). Notably, there are various further theories which consider additional, frequently more controversial and/or purpose-driven views on time, e.g. directedness of time intervals [9, sect. 5.3], branching time approaches, etc., which cannot be covered here. Likewise, we only mention OWL-Time [18], an appendant proposal for the Semantic Web represented in OWL. It is based on a combined point and interval theory axiomatized in FOL that includes Allen's interval relations.

1.2. Time in Top-Level Ontologies

In the context of top-level ontologies, time entities like time points or intervals are usually classified within the corresponding taxonomic structure, but specific theories of time are developed in rare cases (and if so, they often adopt the theories just introduced). Due to spatial limitations, we restrict a closer look to DOLCE [19],[20, mainly ch. 3-4], BFO [21], and PSL [22,23].[5]

Time is included in DOLCE through the category *temporal region*, which is subsumed by the category *abstract* [entity]. Temporal regions are subject to a general, atem-

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[4] Although the axiomatization uses FOL with equality, it is stated that the *equal* relation can be defined by a simple adaptation of one axiom [13, p. 228].

[5] The acronyms stand for Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE), Basic Formal Ontology (BFO), and Process Specification Language (PSL); the respective websites are: `http://www.loa.istc.cnr.it/DOLCE.html`, `http://www.ifomis.org/bfo`, `http://www.mel.nist.gov/psl`.

poral parthood relation. Temporal localization is a special case of quality assignment in DOLCE, involving *temporal qualities* whose values are temporal regions, themselves part of the temporal space. Deliberately, no further assumptions about the temporal space are made in order to remain neutral about ontological commitments on time [19, p. 287].

BFO [20, ch. 7–8], [21] includes the disjoint categories *temporal interval* and *instant* as subcategories of *temporal region* and *occurrent*. Any temporal region is part of *time* (the whole of time). A few initial axioms for time in BFO are available [24],[20, ch. 8]. Axiomatic interval theories do not seem to be included or adapted. Some axioms in [24, sect. 5.1] suggest that time instants are linearly ordered, mereology is applied to temporal regions, and there is a link with the notion of boundaries, as introduced e.g. in [25], by requiring that instants can only exist at the boundary of temporal intervals [24, sect. 5.1].

PSL [22,23] aims at process modeling and is thus related with time. This ontology is highly modularized and completely presented as a FOL axiomatization, with verified consistency for some modules. The core module of the PSL ontology distinguishes *activity*, *activity occurrence*, *object*, and *time-point*; and there is a module for time duration. The relation *before* on time points forms an infinite linear order, with "auxiliary" bounds $+\infty$ and $-\infty$. Density and discreteness are not assumed in the core of PSL, but may be added as an extension. Eventually, we are not aware of any extensions covering intervals and/or interval relations within PSL. However, there are a number of modules axiomatizing mereological, ordering, and duration relations for activities/activity occurrences.

2. Motivating Problems and Requirements for Time in GFO

First of all, an ontology of time should establish a rich basis for analyzing temporal phenomena. In this section we describe three time-related problems with open issues that lead to further demands on a time ontology. These *motivating* problems are reconsidered in section 4, but note that the formalization in section 5 is limited to pure time entities.

2.1. The Holding Problem of Temporal Propositions

The holding problem of temporal propositions (called temporal incidence, e.g. in [26,5]) is concerned with domain-independent conditions that determine the truth-value of propositions through and at times. One aspect of temporal incidence theories is to account for interrelations of propositions holding at different time entities, e.g. *homogeneity*; cf. [27, sect. 2]. Such conditions can be expressed using relations like $holds(\phi, t)$. Its intended meaning is that the proposition ϕ is true at time entity t. We argue that propositions can hold at time intervals and/or at time points. Considering the tossing of a ball (see Figure 1), for example, *the velocity of the ball is zero* holds at one time point, *the ball is raising* holds at an interval (and possibly at many points), *the tossing takes 10 seconds* applies only to an interval.

The current theory of temporal incidence reveals many open problems. Relevant basic notions are insufficiently founded, for example, the notion of an atomic proposition (or elementary sentence). The holding of negation, conjunction and disjunction of temporal propositions is a non-trivial problem, cf. [5, p. 5]. These matters clearly deserve further treatment in future work, while an ontology of time should support the natural expression of temporal incidence conditions and/or the translation of a proposition of a language to an ontologically founded formal sentence.

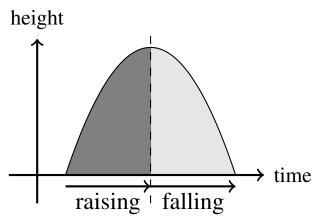

Figure 1. Tossing a ball into the air.

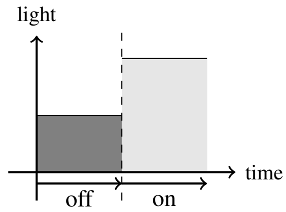

Figure 2. Switching on the light.

2.2. The Dividing Instant Problem

One famous problem involving the holding of propositions shall be given special attention. Allen [14] illustrates the Dividing Instant Problem (DIP) by *switching on the light*, see Figure 2. The central question is whether the light is off or on at the switching point, assuming instantaneous changing from off to on. One might claim that the light is both, off and on, or it is neither off, nor on. Logically, the former leads to an inconsistency, the latter violates the law of excluded middle.

There are several proposals to solve the DIP. Allen excludes instants from the time ontology, and claims that propositions do not satisfy the condition to be true at a time point. We reject this approach due to the implicit reduction of propositions, in the light of the previous section. Another approach stipulates that all intervals are semi-open, e.g. left-closed and right-open [28]. Then, if a proposition holds true throughout a time interval and then holds false throughout a subsequent interval, the truth-value at the dividing instant is false. The weakness of this approach is the arbitrary choice between employing left-closed and right-open vs. left-open and right-closed intervals.

An adequate solution to the DIP can be achieved through satisfying the following conditions, due to supporting consecutiveness of processes without overlap. Note that these cannot be satisfied if we represent time by the ordering of real numbers, the usual understanding of the continuum since the work of *Dedekind* and *Weierstrass*.

1. There are two processes following one another immediately, i.e., without any gaps (the process *light off* meets *light on*)
2. There is a last point t_l in time where the first process ends and there is a first point t_f in time where the second process starts.
3. The points t_l and t_f are distinct.

2.3. Persistence and Change

Another problem, which lacks a comprehensive solution and rests upon a tailored ontology of time, concerns entities that persist through time, though, exhibit different properties at different times. There are several approaches to cope with this problem. David Lewis classifies entities into endurants and perdurants [29]. An entity perdures if it persists by having different temporal parts, or stages, at different times, whereas an entity endures if it persists by being wholly present at any time of its existence. Persistence by endurance is paradoxical and leads to inconsistencies [30]. The stage-approach exhibits serious weaknesses [31]. Both approaches are criticized by various arguments, while further alternatives, e.g. in [32], reveal shortcomings, as well. Consequently, the development of a satisfactory, widely acceptable theory of persistence remains an open problem.

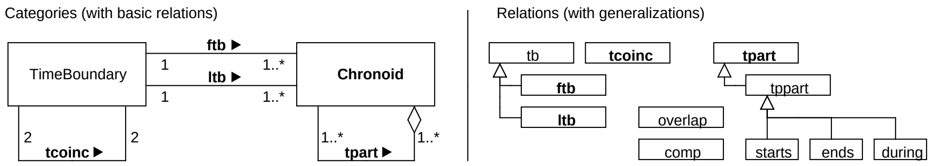

Figure 3. Categories and relations of the time theory of GFO. Boldface signifies primitives.

3. Time in GFO and Basics of the Material Stratum

3.1. $\mathcal{BT}$ – An Ontology of Chronoids, Boundaries and Coincidence

The basic theory of phenomenal time in GFO is abstracted from real-world entities and is inspired by ideas of Franz Brentano [1]; we refer to it as $\mathcal{BT}$. Figure 3 provides an overview of its relevant categories and relations, using for relations the mnemonic predicate names introduced in section 5. Abstract phenomenal time consists of intervals, named *chronoids*, and of *time boundaries*, i.e., time points (roughly speaking). Both are genuine types of entities, where time boundaries depend existentially on chronoids. Only chronoids are subject to *temporal parthood*, and Allen's interval relations (see section 1) apply to them. Every chronoid has exactly two extremal boundaries, which can be understood as the *first* and *last* time point of it. Further, chronoids are truely extended and have infinitely many *inner time boundaries* that arise from proper subchronoids.

An outstanding and beneficial feature of $\mathcal{BT}$ is the relation of *temporal coincidence* between time boundaries, adopted from Brentano. Intuitively, coinciding distinct time boundaries have temporal distance zero. By their means, if a chronoid c_1 *meets* a chronoid c_2, both have distinct extremal boundaries without any overlap or gaps between the last boundary of c_1 and the first of c_2. Time boundaries coincide pairwise. Section 5 captures these and further details of $\mathcal{BT}$ axiomatically, from which it is derivable, among others, that time is linear and unbounded, see section 6.

3.2. Basics of GFO Related to the Material Stratum

A few further remarks on GFO are required before revisiting the motivating problems from above in section 4. First of all, GFO adopts the theory of levels of reality, as expounded by Nicolai Hartmann [4] and Roberto Poli [33]. We restrict the exposition to the *material stratum*, the realm of entities including individuals that are in space and time.

According to their relations to time, individuals are classified into *continuants* (being material endurants), material *presentials* and material *processes*. Processes happen in time and are said to have a temporal extension. Continuants persist through time and have a lifetime, which is a chronoid. A continuant exhibits at any time point of its lifetime a uniquely determined entity, called presential, which is wholly present at the (unique) time boundary of its existence.[6] Examples of continuants are this ball and this tree, being persisting entities with a lifetime. Examples of presentials are this ball and this tree, any

[6]Despite the use of the term 'continuant', this notion is very specific in GFO, for instance, continuants are not wholly present at time boundaries. Note that earlier accounts of this approach made use of different terms, e.g. *abstract substance* [34] and *perpetuant* [2].

of them being wholly present at a certain time boundary t. Hence, the specification of a presential additionally requires the declaration of a time boundary.

In contrast to a presential, a process cannot be wholly present at a time boundary. Examples of processes are particular cases of the tossing of a ball, a 100 m run as well as a surgical intervention, the conduction of a clinical trial, etc. For any process p having the chronoid c as its *temporal extension*, each temporal part[7] of p is determined by taking a temporal part of c and restricting p to this subchronoid. Similarly, p can be restricted to a time boundary t if the latter is a time boundary or an inner boundary of c. The resulting entity is called a *process boundary*, which does not fall into the category of processes.

4. New Modeling Contributions to Temporal Phenomena

4.1. The Holding Problem of Temporal Propositions

Altogether, we propose the ontology $\mathcal{BT}$ as a solid foundation for the analysis of temporal phenomena. It appears immediate from section 3.1 that it provides at least the conceptual means that are known from point-interval theories. The existence of a formal theory interpretation of the theory $\mathcal{IP}$ (and thus $\mathcal{AH}$, see section 1) into $\mathcal{BT}$ provides a formal underpinning to this intuition, see section 6.3. A major novel aspect of $\mathcal{BT}$ is the notion of coincidence of time boundaries.

Coincident time boundaries are important regarding the holding of temporal propositions. They allow for the new case that a proposition holds at a time boundary, but it does not hold at its coincident time boundary. In general, the temporal incidence problem is a special case of a more general problem, namely, to construct a semantic basis for propositions of a language. We hold that such semantic foundation should use an ontological framework, making the content of a linguistic expression explicit, for first steps see [36,37]. This will further involve the notion of truthmakers, present in GFO as facts, being constituents of situations or situoids [35]. Much of this is research in progress, as well as work regarding temporal incidence itself.

4.2. Dividing Instant Problem (DIP)

The DIP has a clear and conclusive solution in $\mathcal{BT}$. We return to the example of switching on the light (Figure 2). The corresponding analysis yields two processes, p, extended over a chronoid with last boundary t_l, and q over a chronoid with first boundary t_f. We may consistently stipulate that all process boundaries of p exhibit the property *light-off*, whereas *light-on* applies to all of q, and t_l and t_f are distinct, but coincide. This exactly satisfies the requirements for a DIP solution in section 2.2.

In this situation there are two properties, contradicting each other, and holding at two different time points having temporal distance zero. This corresponds to our cognition because abstract phenomenal time, exhibiting the phenomenon of coincidence, is accessible introspectively without any metrics, whereas the notion of distance is a result of measuring by using an abstract scale of numbers.

[7] Notably, there are other dimensions by means of which parts of processes can be considered, cf. layers of processes in [35, sect. 8.2.4].

4.3. Persistence and Change

The ontology of time just presented is further among the sources of the GFO approach to persistence. Yet the basic assumption of that approach is grounded on the idea of *integrative realism* [2,3], cf. also [38]. This kind of realism includes the mind as a part of ontology, and postulates a particular relation between the mind and the independent material reality. This relation connects dispositions of a certain type, inhering in the entities of material reality, with the manifold of subjective phenomena occurring in the mind. This relation can be understood as *unfolding the real world disposition X in the mind's medium Y, resulting in the phenomenon Z*. In this relation the mind plays an active role.[8]

In GFO, continuants are viewed as cognitive creations of the mind that possess features of a universal, occuring as the phenomenon of persistence, but also of spatio-temporal individuals, grounded in the presentials which the continuants exhibit. This approach is supported by results of cognitive psychology, notably in Gestalt theory [39].

Continuants may change, because (1) they persist through time and (2) they exhibit different properties at different time boundaries of their lifetime. We hold that only persisting individuals may change. However, a process as a whole cannot change, but it may comprise changes or it may be a change. Hence, *to change* and *to have a change* or *to be a change* are different notions.

Axiom of Object-Process Integration Let c be a continuant. Then there exists a uniquely determined material process, denoted by $Proc(c)$, such that the presentials exhibited by c at the time boundaries of c's lifetime correspond exactly to the process boundaries of $Proc(c)$; cf. [2,35].

This axiom is stipulated for GFO. We say that the continuant c supervenes on the process $Proc(c)$, whose existence is assumed. We hold that a continuant c depends on that process, on the hand, and on the other hand on the mind, since c is supposed – in the framework of GFO – to be cognitively created.

5. Axiomatization of the Ontology $\mathcal{BT}$

Section 3.1 introduces the major notions of the GFO time ontology conceptually (e.g. recall Figure 3). The present section contains the corresponding axiomatic system $\mathcal{BT}$ in first-order predicate logic with equality (FOL), followed by a metalogical analysis in section 6. The set of axioms reflects important properties of chronoids and time boundaries, assuming the domain of discourse is limited to these entities only.[9] The axiom set is not minimal, e.g. axiom A20 is entailed by others. $\mathcal{BT}$ is available[10] in the syntax of the SPASS theorem prover[11] [40], by means of which entailments were checked, including several further consequences, see [41]. We judge almost every axiom on its own to be

[8] The investigation of this type of relation is in its initial stage. It can be associated to the mind-body problem.
[9] Thus one cannot simply form the union of the present theory with other formalized components of GFO, but a corresponding relativization will be required.
[10] `http://www.onto-med.de/ontologies/gfo-time.dfg`
[11] `http://www.spass-prover.org`

easily comprehensible. Therefore, under the spatial limitations, the axioms are presented in the style of a catalog[12] with only short explanatory phrases.

5.1. Basic Signature Elements

B1. $Chron(x)$ (x is a chronoid)
B2. $ftb(x, y)$ (x is the first boundary of y)
B3. $ltb(x, y)$ (x is the last boundary of y)
B4. $tcoinc(x, y)$ (x and y are coincident)
B5. $tpart(x, y)$ (x is a temporal part of y)

5.2. Defined Signature Elements

Relations

D1. $Tb(x) =_{df} \exists y\, tb(x, y)$ (x is a time-boundary)
D2. $TE(x) =_{df} Chron(x) \vee Tb(x)$ (x is a time entity)
D3. $comp(x, y) =_{df} Chron(x) \wedge Chron(y) \wedge \exists z(Chron(z) \wedge tpart(x, z) \wedge tpart(y, z))$ (x and y are compatible chronoids)
D4. $during(x, y) =_{df} Chron(x) \wedge Chron(y) \wedge tppart(x, y) \wedge \neg starts(x, y) \wedge \neg ends(x, y)$ (Allen's "during" relation [14])
D5. $ends(x, y) =_{df} Chron(x) \wedge Chron(y) \wedge tppart(x, y) \wedge \exists u(ltb(u, x) \wedge ltb(u, y))$ (Allen's "ends" relation [14])
D6. $meets(x, y) =_{df} Chron(x) \wedge Chron(y) \wedge \exists uv(ltb(u, x) \wedge ftb(v, y) \wedge tcoinc(u, v))$ (Allen's "meets" relation [14])
D7. $starts(x, y) =_{df} Chron(x) \wedge Chron(y) \wedge tppart(x, y) \wedge \exists u(ftb(u, x) \wedge ftb(u, y))$ (Allen's "starts" relation [14])
D8. $tb(x, y) =_{df} ftb(x, y) \vee ltb(x, y)$ (x is a time boundary of y)
D9. $tov(x, y) =_{df} \exists z(tpart(z, x) \wedge tpart(z, y))$ (temporal overlap of chronoids)
D10. $tppart(x, y) =_{df} tpart(x, y) \wedge x \neq y$ (proper temporal part-of)

Functions

D11. $ft(x) = y \leftrightarrow_{df} ftb(y, x)$ (functional ftb: the first time boundary of x is y)
D12. $lt(x) = y \leftrightarrow_{df} ltb(y, x)$ (functional ltb: the last time boundary of x is y)

5.3. Axioms

Taxonomic Axioms

A1. $TE(x)$ (the domain of discourse covers only time entities)
A2. $Chron(x) \wedge Chron(y) \rightarrow comp(x, y)$ (every two chronoids are compatible)
A3. $\neg \exists x(Chron(x) \wedge Tb(x))$ (chronoid and time boundary are disjoint categories)

[12]The signature splits into basic and defined symbols, where the latter are introduced together with their definitions. These two parts are sorted by the arity of symbols and alphabetically for the same arity, for better reference, instead of building on one another consecutively. The actual axioms are arranged by adopting the complexity of formulas and mutual relations regarding content as guiding aspects, which also leads to the grouping into three types. Finally, note that all formulas are implicitly universally quantified.

A4. $tpart(x,y) \rightarrow Chron(x) \wedge Chron(y)$ (temporal part-of is a relation on chronoids)

A5. $tcoinc(x,y) \rightarrow Tb(x) \wedge Tb(y)$ (coincidence is a relation on time boundaries)

A6. $tb(x,y) \rightarrow Tb(x) \wedge Chron(y)$ (tb relates time boundaries with chronoids)

Structure of single relations

A7. $Chron(x) \rightarrow tpart(x,x)$ (reflexivity)

A8. $tpart(x,y) \wedge tpart(y,x) \rightarrow x = y$ (antisymmetry)

A9. $tpart(x,y) \wedge tpart(y,z) \rightarrow tpart(x,z)$ (transitivity)

A10. $Chron(x) \rightarrow \exists y(starts(x,y))$ (every chronoid has a future extension)

A11. $Chron(x) \rightarrow \exists y(ends(x,y))$ (every chronoid has a past extension)

A12. $Chron(x) \rightarrow \exists y(during(y,x))$ (during every chronoid there is another one)

A13. $Chron(x) \rightarrow \exists y(ftb(y,x))$ (every chronoid has a first boundary)

A14. $Chron(x) \rightarrow \exists y(ltb(y,x))$ (every chronoid has a last boundary)

A15. $Chron(x) \wedge ftb(y,x) \wedge ftb(z,x) \rightarrow y = z$ (the first boundary of chronoids is unique)

A16. $Chron(x) \wedge ltb(y,x) \wedge ltb(z,x) \rightarrow y = z$ (the last boundary of chronoids is unique)

A17. $Tb(x) \rightarrow \exists y(tb(x,y))$ (every time boundary is a boundary of a chronoid)

A18. $Tb(x) \rightarrow tcoinc(x,x)$ (reflexivity)

A19. $tcoinc(x,y) \rightarrow tcoinc(y,x)$ (symmetry)

A20. $tcoinc(x,y) \wedge tcoinc(y,z) \rightarrow tcoinc(x,z)$ (transitivity)

A21. $Tb(x) \rightarrow \exists y(x \neq y \wedge tcoinc(x,y))$ (every time boundary coincides with another one)

A22. $tcoinc(x,y) \wedge tcoinc(x,z) \rightarrow x = y \vee x = z \vee y = z$
(at most two distinct time boundaries coincide)

Interaction axioms

A23. $tov(x,y) \rightarrow$ (two overlapping chronoids have an intersection)
$\exists z(tpart(z,x) \wedge tpart(z,y) \wedge \forall u(tpart(u,x) \wedge tpart(u,y) \rightarrow tpart(u,z)))$

A24. $Chron(x) \wedge Chron(y) \wedge \neg tpart(x,y) \rightarrow \exists z(tpart(z,x) \wedge \neg tov(z,y))$
(where one chronoid is not a part of another one, there exists a non-overlapping part)

A25. $Chron(x) \wedge Chron(y) \wedge tcoinc(ft(x), ft(y)) \wedge tcoinc(lt(x), lt(y)) \rightarrow x = y$
(there are no distinct chronoids with coincident boundaries)

A26. $tcoinc(x,y) \rightarrow \neg\exists w((ftb(x,w) \wedge ltb(y,w)) \vee (ltb(x,w) \wedge ftb(y,w)))$
(coincident boundaries are boundaries of distinct chronoids)

A27. $Tb(x) \wedge Tb(y) \wedge \neg tcoinc(x,y) \rightarrow$
$\exists z(Chron(z) \wedge$
$((tcoinc(x, ft(z)) \wedge tcoinc(y, lt(z))) \vee (tcoinc(x, lt(z)) \wedge tcoinc(y, ft(z)))))$
(between any two non-coincident time boundaries there is a corresponding chronoid)

A28. $tcoinc(x,y) \wedge ftb(x,u) \wedge ftb(y,v) \rightarrow (tpart(u,v) \vee tpart(v,u))$
(coincident first boundaries entail parthood)

A29. $tcoinc(x,y) \wedge ltb(x,u) \wedge ltb(y,v) \rightarrow (tpart(u,v) \vee tpart(v,u))$
(coincident last boundaries entail parthood)

A30. $Chron(x) \wedge Chron(y) \wedge \exists u(Chron(u) \wedge ft(u) = ft(y) \wedge tcoinc(lt(u), ft(x))) \wedge$
$\exists v(Chron(v) \wedge tcoinc(ft(v), lt(x)) \wedge lt(v) = lt(y)) \quad \rightarrow \quad during(x,y)$
(x is during y if embedded between two chronoids with appropriate boundaries)

A31. $tpart(x,y) \wedge ft(x) \neq ft(y) \rightarrow \exists z(starts(z,y) \wedge tcoinc(lt(z), ft(x)))$
(for every part with distinct first boundaries there is a corresponding starts-fragment)

A32. $tpart(x,y) \wedge lt(x) \neq lt(y) \rightarrow \exists z(ends(z,y) \wedge tcoinc(ft(z), lt(x)))$
(for every part with distinct end boundaries there is a corresponding ends-fragment)

A33. $Chron(x) \wedge Chron(y) \wedge meets(x,y) \rightarrow$ (the sum of meeting chronoids is a chronoid)
$\exists z(Chron(z) \wedge ft(x) = ft(z) \wedge lt(y) = lt(z) \wedge$
$\neg\exists u(tpart(u,z) \wedge \neg tov(u,x) \wedge \neg tov(u,y)))$

A34. $tov(x,y) \wedge \exists w(starts(w,x) \wedge \neg tov(w,y)) \rightarrow$ (the sum of overlapping chronoids is a chronoid)
$\exists z(Chron(z) \wedge ft(x) = ft(z) \wedge lt(y) = lt(z) \wedge$
$\neg\exists u(tpart(u,z) \wedge \neg tov(u,x) \wedge \neg tov(u,y)))$

6. Metalogical Analyses

The development of $\mathcal{BT}$ is grounded on the axiomatic method, established by David Hilbert [42], and systematically introduced as a basic task of formal ontology in, e.g., [3, sect. 2]. This method additionally includes the metalogical investigation of the considered theory. Consistency is central in this respect, but not the only issue. In this section we summarize our metalogical results for $\mathcal{BT}$.[13]

6.1. Linearity of Time

It appears to be intuitive that time is associated with a linear ordering that should be established on the basis of the theory $\mathcal{BT}$. Such a linear ordering can be defined on the set of equivalence classes of time boundaries. According to the axioms (A18–A22), the coincidence relation as defined on time boundaries is an equivalence relation whose equivalence classes contain exactly two elements. Let $[x]$ be the equivalence class corresponding to the time boundary x. We introduce a relation $\prec$ ('before') on coincidence classes of time boundaries that is defined by the following condition: $[x] \prec [y]$ iff there exists a chronoid whose first boundary belongs to $[x]$ and whose last boundary belongs to $[y]$.

Proposition 1. *Let $\mathcal{A}$ be a model of $\mathcal{BT}$, and let $[Tb]$ the set of coincidence classes of time boundaries. Then the relation $\prec$ defines a dense linear ordering (without least and without greatest element) on the set $[Tb]$.*

6.2. Consistency, Completeness and Decidability of the Theory $\mathcal{BT}$

A proof of (relative) consistency of $\mathcal{BT}$ can be achieved by reduction to the monadic second order theory of linear orderings. Let λ be the order type of the real numbers $\mathbb{R}$. We construct a new linear ordering $\mathcal{M} = (M, \leq)$, the type of which is denoted by $\lambda(2)$, by taking the linear ordered sum over $\mathbb{R}$ of linear orderings of type $\mathbf{2}$. Within the monadic second order theory of $\mathcal{M}$ we define the relations $Chron(x)$, $ftb(x,y)$, $ltb(x,y)$, $tcoinc(x,y)$, $tpart(x,y)$ as follows. $Chron(x)$ if and only if x is an interval $[a,b]$ over M whose first element a has an immediate predecessor, and whose last element b has an immediate successor. The first and the last boundary of a chronoid is determined by the corresponding elements of the associated interval. Two distinct boundaries coincide if one is an immediate successor of the other. Eventually, temporal part-of corresponds to the subinterval relation. In this way we get a new structure $\mathcal{A}$, for which it easily follows that $\mathcal{A}$ satisfies all axioms of $\mathcal{BT}$.

[13] Formal proofs of all propositions are available in the extended report [41].

Proposition 2. *The theory $\mathcal{BT}$ is consistent.*

Further metalogical results on $\mathcal{BT}$ concern its completeness and decidability. The latter means that there is an effective method to decide whether a sentence is a consequence of $\mathcal{BT}$, cf. e.g. [43] for these notions.

Proposition 3. *The theory $\mathcal{BT}$ is complete and decidable.*

The proof relies on showing that $\mathcal{BT}$ is an ω-categorical theory, i.e., any two countable models of $\mathcal{BT}$ are isomorphic. We define a corresponding isomorphism between two arbitrary models of $\mathcal{BT}$, starting from an isomorphism between their sets of coincidence classes. The latter is justified by Proposition 1 and a theorem of Georg Cantor that any two countable dense linear orderings are isomorphic [43, p. 149]. ω-categoricity of $\mathcal{BT}$ entails its completeness. Eventually, completeness and axiomatizability of a theory yield its decidability [43, p. 147].

6.3. Relationship with Established Time Theories

Remembering the available work on time, cf. section 1, it is of interest to relate $\mathcal{BT}$ to other axiomatizations. It is rather straightforward to show that $\mathcal{BT}$ covers $\mathcal{IP}_{\text{dense}}$ [17,5] (and thus Allen and Hayes well-known theory, as well, see section 1).

Proposition 4. *The point-interval theory $\mathcal{IP}_{dense}$ is interpretable in $\mathcal{BT}$.*

This theory interpretation, cf. [43, sect. 2.7], naturally links points$^{\mathcal{IP}}$ and time boundaries$^{\mathcal{BT}}$, and intervals$^{\mathcal{IP}}$ and chronoids$^{\mathcal{BT}}$. This results implicitly from mapping the *before*$^{\mathcal{IP}}$ relation between points to $before^{\mathcal{BT}}$,[14] and *begin*$^{\mathcal{IP}}(x,y)$ and *end*$^{\mathcal{IP}}(x,y)$ by the formulas $\exists z(tcoinc(x,z) \wedge ftb(z,y))$ and $\exists z(tcoinc(x,z) \wedge ltb(z,y))$, respectively. The latter cases indicate that temporal coincidence requires some care in the interpretation. Unconventionally, equality of points$^{\mathcal{IP}}$ must be interpreted by $tcoinc^{\mathcal{BT}}$. This interpretation connects each point$^{\mathcal{IP}}$ with a unique time boundary$^{\mathcal{BT}}$, such that distinct points$^{\mathcal{IP}}$ yield non-coincident time boundaries$^{\mathcal{BT}}$. Importantly and although all of this nicely matches the intuitions behind the two theories, theory interpretation is a formal tool for analyzing theory interrelations. It should not be understood to provide ontological insights across the theories under consideration.

7. Conclusions and Future Work

In this paper we presented a new approach to phenomenal time, which is adopted by the top-level ontology GFO [2]. This time ontology, $\mathcal{BT}$ herein, is inspired by ideas of Franz Brentano [1]. Its basic concepts are chronoids and time boundaries (also called time points); the basic relations are *temporal part-of*, temporal *coincidence of time boundaries*, and two relations of *being a time boundary of a chronoid*. This ontology is formalized by a set of axioms, specifying logical links between the concepts and the relations. It is the basis for the development of a comprehensive ontology of material entities, in-

[14] $before^{\mathcal{BT}}$ arises from a definitional extension of $\mathcal{BT}$ with a predicate reflecting the relation < in section 6.1: $before(x,y) =_{df} \exists uvz(Chron(z) \wedge ftb(u,z) \wedge ltb(v,z) \wedge tcoinc(u,x) \wedge tcoinc(v,y))$.

cluding continuants and processes. $\mathcal{BT}$ allows for, among others, a consistent and conclusive solution to the Dividing Instant Problem. We believe that the temporal continuum can be introspectively accessed without any metrics, and that it cannot be understood and grasped by the set of its points. Moreover, we expect a series of new applications, and have already verified the usefulness of our ontology as a tool for modeling cellular genealogies [44] and surgical interventions [37], for instance.

There is a number of open problems that can be classified into logical, ontological, and semantical problems. The theory $\mathcal{BT}$ can be further analyzed and extended in several directions. One open issue is to find minimal sets of axioms, such that each axiom is independent of the remaining theory. Furthermore, it might be useful to extend the temporal domain by adding time regions, which can be considered as mereological sums of chronoids. Such time regions are, in general, not connected. Hence, the axioms must be modified and will become more complicated. The development and metalogical analyses of extended systems are work in progress. The main semantical problem concerns the temporal incidence problem for propositions. We expect that an expressive ontology of truthmakers must be established as a semantic basis for the interpretation of temporal propositions.

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